BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

CSI 30 — Take Home Exam
Spring 2024 — Day: 05/01/2024

Print Name: ____

1. Given the algorithm:

   procedure partial($a_1, a_2, a_3, \ldots, a_n$: integers)
   $prod_1 := 1$
   $prod_2 := 1$
   for i := 1 to n
     if ($a_i > 0$): $prod_1 := prod_1 \cdot a_i$
     if ($a_i < 0$): $prod_2 := prod_2 \cdot a_i$
   return($prod_1, prod_2$)

   For the set of values -6,2,-3,-8,10,3,-1 as input for the above algorithm, what are values of $prod_1$ and $prod_2$ that will be returned?

2. Describe the steps in the binary search to find the location of 9 in the following list: $\{1, 6, 8, 9, 13, 14, 16, 22, 36, 38\}$.

3. Use bubble sort to put the list $\{7, 3, 5, 1, 2\}$ into increasing order. How many comparison were performed?

4. Use insertion sort to put the list $\{7, 3, 5, 1, 2\}$ into increasing order. How many comparison were performed?

5. Find the greatest common divisor GCD$(888, 54)$ using the Euclidean Algorithm:

   procedure GCD (a, b: positive integers):
   x := a
   y := b
   While $y \neq 0$:
     r := x mod y
     x := y
     y := r
   return('The GCD is' : x)

   Find integers t, s such that $888t + 54s =$ GCD(888, 54).

6. Find the expansion of 211 in base 5.

7. Find the base 16 expansion of 211. (Use the symbols A = 10, B = 11, ..., F = 15.)

8. Compute the decimal representation of $(17561)_8$.

9. Compute the decimal representation of $(12201)_3$.

10. Compute, without a calculator $(98 \cdot 102) \bmod 100$.

11. Find, without a calculator $4^{100} \bmod 5$.

12. Find, without a calculator, the last digit of 3^{110}.

13. Show that the equation:
$$x^2 + y^2 = 4z + 3$$
has no solutions $x, y, z \in \mathbb{Z}$. Hint: Use congruence mod 4.

14. Let $x, y \in \mathbb{N}$ be relatively prime. If the product xy is a perfect square, prove that x and y must both be perfect squares. Hint: use the Fundamental Theorem of Arithmetic.